

Higgs G-inflation

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A new class of inflation models within the context of G-inflation is proposed, in which the standard model Higgs boson can act as an inflaton thanks to Galileon-like non-linear derivative interaction. The generated primordial density perturbation is shown to be consistent with the present observational data. We also make a general discussion on potential-driven G-inflation models, and find a new consistency relation between the tensor-to-scalar ratio r and the tensor spectral index n_T , $r = -32\sqrt{6}n_T/9$, which is crucial in discriminating the present models from standard inflation with a canonical kinetic term.

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I. INTRODUCTION

Primordial inflation [1, 2] is now regarded as a part of the “standard” cosmology because it not only solves the flatness and the horizon problems but also accounts for the origin of primordial fluctuations [3]. To construct a model of inflation, one usually assumes a scalar field that drives inflation (called an inflaton) outside the standard model (SM) of particle physics. This is because there are no scalar fields in the SM except for the Higgs boson and it has been found that the SM Higgs boson cannot be responsible for inflation as long as its kinetic term is canonical and it is minimally coupled to gravity [4]. The difficulty here lies in the fact that the self interaction of the SM Higgs boson is so strong that the resultant primordial density fluctuation would be too large to be consistent with the present observational data [5].

To construct inflation models within the SM, several variants of Higgs-driven inflation have been proposed so far. They include models with a non-minimal coupling term to gravity [6] and with a non-minimal coupling of the Higgs kinetic term with the Einstein tensor [7].¹ The amplitude of the curvature perturbation is suppressed due to the large effective Planck scale in the former case, while in the latter case the same thing is caused by the enhanced kinetic function which effectively reduce the self coupling of the Higgs boson.

The simplest way to enhance the kinetic energy would be to add a non-canonical higher order kinetic term. A number of novel inflation models with non-standard kinetic terms have been proposed, such as k-inflation [9], ghost condensate [10], and Dirac-Born-Infeld inflation [11]. When incorporating higher order kinetic terms special care must be taken in order to avoid unwanted ghost instabilities. Since newly introduced degrees of freedom will lead easily to ghosts, it would be desirable if the scalar field does not give rise to a new degree of freedom in spite of its higher derivative nature. It has recently been shown that special combinations of higher order kinetic terms in the Lagrangian produce derivatives no higher than two both in the gravitational and scalar field equations [12, 13]. A scalar field having this property is often called the Galileon because it possesses a Galilean shift symmetry in the Minkowski background. Such a scalar field has been studied in the context of modified gravity and dark energy in [14]. Recently, an inflation model dubbed as “G-inflation” was proposed [15], in which inflation is driven by a scalar field with a Galileon-like kinetic term. In Ref. [15], the background and perturbation dynamics of G-inflation were investigated, revealing interesting features brought by the Galileon term. For example, scale-invariant scalar perturbations can be generated even in the exactly de Sitter background, and the tensor-to-scalar ratio can take a significantly larger value than in the standard inflation models, violating the standard consistency relation. Other aspects of G-inflation have been explored in Refs. [16, 17] (see also [18]).

In this paper, we propose a new Higgs inflation model by adding a Galileon-like kinetic term to the standard Higgs Lagrangian. We show that a self coupling constant of the order of the unity is compatible with the present observational data thanks to the kinetic term enhanced by the Galileon effect. We however start with a general discussion on G-inflation driven by the poten-

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¹ Inflation models where Higgs multiplets act as an inflaton in the context of supersymmetric extensions of the SM, *e.g.*, the next-to-minimal supersymmetric standard model, have also been proposed in [8]. In these models, a non-canonical Kähler potential for the Higgs multiplet is assumed.

tial term because our potential-driven G-inflation is not restricted only to the Higgs field. We first give a criterion to determine which term becomes dominant in the kinetic term. Then, the slow-roll parameters and the slow-roll conditions are concretely given in terms of the potential $V(\phi)$ and the function characterizing the Galileon term. We also derive the expressions for primordial fluctuations in terms of the slow-roll parameters, and find a new model-independent consistency relation for a potential-driven G-inflation model, which is quite useful for discriminating it from the standard inflation model with a canonical kinetic term. It turns out, however, that primordial non-Gaussianity of the curvature fluctuation is not large in potential-driven G-inflation. Finally, as a concrete example of a potential-driven G-inflation model, we propose a Higgs G-inflation model. This model predicts that the scalar spectral index $n_s \simeq 0.967$ and the tensor-to-scalar ratio $r \simeq 0.14$ for the number of e -folds $\mathcal{N} = 60$, which, together with the new consistency relation $r = -32\sqrt{6}n_T/9$, makes our Higgs G-inflation model testable in near future.

This paper is organized as follows. In the next section, we make a general discussion on the potential-driven G-inflation model. In Sec. III, we apply it to more concrete examples, which have chaotic-type, new-type, and hybrid type potential forms. In Sec. IV, we present a new class of inflation model that regards the standard model Higgs boson as an inflaton in the context of G-inflation. Final section is devoted to conclusions and discussion.

II. POTENTIAL-DRIVEN G-INFLATION

The general Lagrangian describing lowest-order G-inflation is of the form [15]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + K(\phi, X) - G(\phi, X) \square \phi \right], \quad (1)$$

where M_{Pl} is the reduced Planck mass, R is the Ricci scalar and $X := -\nabla_\mu \phi \nabla^\mu \phi / 2$. The main focus of the present paper is G-inflation driven by the potential term with the kinetic term modified by the $G(\phi, X)$ term. We therefore take the “standard” form of the function $K(\phi, X)$,

$$K(\phi, X) = X - V(\phi), \quad (2)$$

while for simplicity we assume the following form of the $G(\phi, X)$ term,

$$G(\phi, X) = -g(\phi)X. \quad (3)$$

A. The background dynamics

Taking the homogeneous and isotropic metric $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$, we have the following basic equations

governing the background cosmological dynamics:

$$3M_{\text{Pl}}^2 H^2 = X \left[1 - gH\dot{\phi}(6 - \alpha) \right] + V, \quad (4)$$

$$M_{\text{Pl}}^2 \dot{H} = -X \left[1 - gH\dot{\phi}(3 + \eta - \alpha) \right], \quad (5)$$

$$H\dot{\phi} \left[3 - \eta - gH\dot{\phi}(9 - 3\epsilon - 6\eta + 2\eta\alpha) \right] + (1 + 2\beta)V' = 0, \quad (6)$$

where the dot represents derivative with respect to t and the prime with respect to ϕ . In the above we have defined

$$\epsilon := -\frac{\dot{H}}{H^2}, \quad (7)$$

$$\eta := -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad (8)$$

$$\alpha := \frac{g'\dot{\phi}}{gH}, \quad (9)$$

$$\beta := \frac{g''X^2}{V'}. \quad (10)$$

We assume that all of these quantities are small:

$$\epsilon, |\eta|, |\alpha|, |\beta| \ll 1. \quad (11)$$

The condition $|\alpha| \ll 1$ indicates that $g(\phi(t))$ must be a slowly-varying function of time. Equations (4) and (5) together with these slow-roll conditions imply

$$X, |gH\dot{\phi}X| \ll V. \quad (12)$$

Thus, the energy density is dominated by the potential V under the slow-roll conditions:

$$3M_{\text{Pl}}^2 H^2 \simeq V. \quad (13)$$

The slow-roll equation of motion for the scalar field is given by

$$3H\dot{\phi} \left(1 - 3gH\dot{\phi} \right) + V' \simeq 0. \quad (14)$$

One can consider two different limiting cases here. The case $|gH\dot{\phi}| \ll 1$ corresponds to standard slow-roll inflation, while in the opposite limit, $|gH\dot{\phi}| \gg 1$, the Galileon effect alters the scalar field dynamics. We are interested in the latter case. Since $9H^2\dot{\phi}^2 \simeq V'/g$, it is required that $V'/g > 0$ in order for this regime to be realized. The slow-roll equation of motion can be solved for $\dot{\phi}$ to give

$$\dot{\phi} \simeq -\text{sgn}(g)M_{\text{Pl}} \left(\frac{V'}{3gV} \right)^{1/2}. \quad (15)$$

We have fixed the sign of $\dot{\phi}$ so that $\text{sgn}(\dot{\phi}) = -\text{sgn}(V')$, i.e., the scalar field rolls *down* the potential. This seems to be a natural situation for the scalar field dynamics. As we will see below, ghost instabilities are avoided provided

that $g\dot{\phi} < 0$, and hence only in this branch the Universe can be stable. From Eq. (15) we see

$$-gH\dot{\phi} \simeq \frac{1}{3}(gV')^{1/2}. \quad (16)$$

Therefore, the condition that the kinetic term coming from $G(\phi, X)$ is much bigger than the usual linear kinetic term X is equivalent to

$$gV' \gg 1. \quad (17)$$

Using the slow-roll equations, one can rewrite the slow-roll parameters in terms of the potential as

$$\epsilon \simeq \epsilon_{\text{std}} \frac{1}{(gV')^{1/2}}, \quad (18)$$

$$\eta \simeq \frac{\tilde{\eta}_{\text{std}}}{2} \frac{1}{(gV')^{1/2}} - \epsilon + \frac{\alpha}{2}, \quad (19)$$

$$\alpha \simeq -M_{\text{Pl}}^2 \frac{g'}{g} \frac{V'}{V} \frac{1}{(gV')^{1/2}}, \quad (20)$$

$$\beta \simeq \frac{M_{\text{Pl}}^4}{36} \frac{g''}{g} \left(\frac{V'}{V} \right)^2 \frac{1}{gV'}, \quad (21)$$

where ϵ_{std} and $\tilde{\eta}_{\text{std}}$ are the slow-roll parameters conventionally used for standard slow-roll inflation,

$$\epsilon_{\text{std}} := \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2, \quad \tilde{\eta}_{\text{std}} := M_{\text{Pl}}^2 \frac{V''}{V}. \quad (22)$$

Equations (18) and (19) clearly show that the Galileon term effectively flattens the potential thanks to the factor $1/(gV')^{1/2}$ ($\ll 1$). This implies that in the presence of the Galileon-like derivative interaction slow-roll inflation can take place even if the potential is rather steep.

For later convenience we define $\tilde{\eta} := \tilde{\eta}_{\text{std}}/2(gV')^{1/2}$. It will be also useful to note that

$$\frac{g\dot{\phi}^3}{M_{\text{Pl}}^2 H} \simeq -\frac{2}{3}\epsilon. \quad (23)$$

This means that even if the Galileon dominates the dynamics of slow-roll inflation, the standard part of the Lagrangian remains much larger than the Galileon interaction term,

$$|K(\phi, X)| \simeq V(\phi) \gg |G(\phi, X)\Box\phi|. \quad (24)$$

Let us make a brief comment on the initial condition for the scalar field. The field may initially be off along the slow-roll trajectory (15). As long as $\text{sgn}(\dot{\phi}) = -\text{sgn}(g)$, the field safely approaches the trajectory (15). If $\text{sgn}(\dot{\phi}) = +\text{sgn}(g)$ initially, the situation is more subtle, because the solution would approach another branch of the slow-roll attractor and the field would go on to climb up the potential. This is what indeed happens if $\mathcal{F}, \mathcal{G} < 0$ at the initial moment, signaling ghost instabilities [see Eqs. (28)–(30) below]. Note, however, that in Eq. (30) \mathcal{F} and \mathcal{G} are evaluated along the slow-roll trajectory; it is therefore possible in principle that $\text{sgn}(\dot{\phi}) = +\text{sgn}(g)$ but still one has $\mathcal{F}, \mathcal{G} > 0$ at the initial moment. In this case the solution approaches the healthy branch of the slow-roll attractor.

B. Primordial fluctuations

Let us investigate the properties of scalar cosmological perturbations in potential-dominated G-inflation. The quadratic action for the curvature perturbation in the unitary gauge, \mathcal{R} , is given by [15]

$$S_2 = M_{\text{Pl}}^2 \int d\tau d^3x a^2 \sigma \left[\frac{1}{c_s^2} (\partial_\tau \mathcal{R})^2 - (\vec{\nabla} \mathcal{R})^2 \right], \quad (25)$$

where τ is the conformal time and

$$\sigma := \frac{X\mathcal{F}}{M_{\text{Pl}}^2 \left(H - \dot{\phi} X G_X / M_{\text{Pl}}^2 \right)^2}, \quad (26)$$

$$c_s^2 := \frac{\mathcal{F}}{\mathcal{G}}, \quad (27)$$

with

$$\begin{aligned} \mathcal{F} := & K_X + 2G_X(\ddot{\phi} + 2H\dot{\phi}) - 2\frac{G_X^2}{M_{\text{Pl}}^2} X^2 \\ & + 2G_{XX}X\ddot{\phi} - 2(G_\phi - XG_{\phi X}), \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{G} := & K_X + 2XK_{XX} + 6G_XH\dot{\phi} + 6\frac{G_X^2}{M_{\text{Pl}}^2} X^2 \\ & - 2(G_\phi + XG_{\phi X}) + 6G_{XX}HX\dot{\phi}. \end{aligned} \quad (29)$$

The above expressions are for general $K(\phi, X)$ and $G(\phi, X)$, but in the present case we simply have

$$\mathcal{F} \simeq -4gH\dot{\phi}, \quad \mathcal{G} \simeq -6gH\dot{\phi}, \quad (30)$$

and hence

$$\sigma \simeq \frac{4}{3}\epsilon, \quad c_s^2 \simeq \frac{2}{3}, \quad (31)$$

where we used Eq. (23). Note that $g\dot{\phi} < 0$ is required to ensure the stability against perturbations, as seen from Eq. (30)

Evaluating the power spectrum from the quadratic action (25) is a standard exercise; we arrive at

$$\begin{aligned} \mathcal{P}_{\mathcal{R}} &= \frac{1}{4\pi^2} \frac{H^2}{2\sigma c_s M_{\text{Pl}}^2} \Big|_{\tau=1/c_s k} \\ &= \frac{3\sqrt{6}}{64\pi^2} \frac{H^2}{M_{\text{Pl}}^2 \epsilon} \Big|_{\tau=1/c_s k}. \end{aligned} \quad (32)$$

The spectral tilt, $n_s - 1 = d \ln \mathcal{P}_{\mathcal{R}} / d \ln k$, can be evaluated as

$$n_s - 1 = -6\epsilon + 3\tilde{\eta} + \frac{\alpha}{2}, \quad (33)$$

where the relation $\ddot{H}/H\dot{H} = -\epsilon - 3\eta + \alpha$ was used.

The tensor perturbations are generated in the same way as in the usual canonical inflation models, and hence

the power spectrum and the spectral index of the primordial gravitational waves are given by

$$\mathcal{P}_T = \frac{8}{M_{\text{Pl}}^2} \left(\frac{H}{2\pi} \right)^2 \Big|_{\tau=1/k}, \quad n_T = -2\epsilon. \quad (34)$$

Thus, we obtain a new, model-independent consistency relation between the tensor-to-scalar ratio and the tensor spectral index:

$$r = 16\sigma_{c_s} = -\frac{32\sqrt{6}}{9}n_T. \quad (35)$$

III. GALILEAN SYMMETRIC MODELS

In this section, we shall clarify slow-roll dynamics of G-inflation for three representative forms of the potential. We consider the simplest case where the Galileon-type kinetic term respects not only the Galilean shift symmetry in the Minkowski background, but also the shift symmetry $\phi \rightarrow \phi + \text{const}$ [19] during inflation, *i.e.*,

$$|g| = \frac{1}{M^3} = \text{const.}, \quad (36)$$

where M is a mass scale. Here the sign of g should be chosen to coincide with that of $V'(\phi)$. For (36) the G term is Z_2 odd, and hence the slow-roll solution (15) can be realized only in one side of a Z_2 -symmetric potential. Note, however, that the following results can be generalized qualitatively to the cases with more general g having weak dependence on ϕ , because $g(\phi)$ may be practically constant for slowly-rolling ϕ . Note also that we assume (36) only in the inflationary stage; g may change globally in ϕ -space and the detailed shape of $g(\phi)$ would play an important role during the reheating stage after inflation. From the conservative point of view, reheating will proceed in the same way as in the usual inflation models by taking $g(\phi)$ such that $g(\phi) \rightarrow 0$ around the minimum of the potential $V(\phi)$. In this section we focus on the dynamics of ϕ in the inflationary stage, and we will come back to the issue of reheating in Sec. IV.

A. Chaotic inflation

First, let us consider the chaotic inflation model [19, 20] for which the potential is given by

$$V(\phi) = \frac{\lambda}{n} \phi^n, \quad (37)$$

with $n (\geq 1)$ being an integer. We assume that the field is moving in the $\phi > 0$ side and hence $g = +1/M^3 > 0$. In this case, the condition $gV' \gg 1$ is equivalent to

$$\phi \gg \left(\frac{3M^{3/2}}{\lambda^{1/2}} \right)^{2/(n-1)} =: \phi_G. \quad (38)$$

Since the slow-roll parameters for $\phi \gg \phi_G$ are given by

$$\epsilon = \frac{n^2 M_{\text{Pl}}^2 M^{3/2}}{2\lambda^{1/2} \phi^{(n+3)/2}}, \quad \tilde{\eta} = \frac{n-1}{n} \epsilon, \quad (39)$$

potential-driven G-inflation proceeds as long as

$$\phi \gg \left(\frac{n^2 M_{\text{Pl}}^2 M^{3/2}}{2\lambda^{1/2}} \right)^{2/(n+3)} =: \phi_{\epsilon G}. \quad (40)$$

If $\phi_G > \phi_{\epsilon G}$, one can consider the scenario in which standard chaotic inflation follows slow-roll G-inflation. This scenario is possible if

$$M > \frac{n^{(n-1)/3}}{2^{(n-1)/6} 3^{(n+3)/6}} \lambda^{1/3} M_{\text{Pl}}^{(n-1)/3} =: M_c. \quad (41)$$

If, on the other hand, $\phi_G < \phi_{\epsilon G}$, *i.e.*, $M < M_c$, slow-roll G-inflation ends at $\phi = \phi_{\epsilon G}$ and standard chaotic inflation does not follow. In this case, G-inflation is possible even in the region where the potential is too steep to support standard chaotic inflation. In this case, the number of e -folds \mathcal{N} reads

$$\mathcal{N} = \int_{\phi}^{\phi_{\epsilon G}} \frac{H}{\dot{\phi}} d\phi = \frac{2\lambda^{1/2}}{n(n+3)M_{\text{Pl}}^2 M^{3/2}} \phi^{(n+3)/2} - \frac{n}{n+3}. \quad (42)$$

From this we obtain the field value evaluated \mathcal{N} e -folds before the end of inflation,

$$\phi_{\mathcal{N}} = [(n+3)\mathcal{N} + n]^{2/(n+3)} \left(\frac{nM_{\text{Pl}}^2 M^{3/2}}{2\lambda^{1/2}} \right)^{2/(n+3)}. \quad (43)$$

The situation is summarized in Fig. 1.

Now we investigate the primordial perturbation. In the present case we find

$$\mathcal{F} \simeq \frac{4}{3} \frac{\lambda^{1/2} \phi^{(n-1)/2}}{M^{3/2}}, \quad \mathcal{G} \simeq \frac{2\lambda^{1/2} \phi^{(n-1)/2}}{M^{3/2}}. \quad (44)$$

From Eqs. (32) and (33), the primordial density perturbation generated during the potential-dominated chaotic G-inflation is evaluated as

$$\begin{aligned} \mathcal{P}_{\mathcal{R}} &\simeq \frac{\sqrt{6}}{32\pi^2 n^3} \frac{\lambda^{3/2} \phi^{3(n+1)/2}}{M^{3/2} M_{\text{Pl}}^6} \\ &\simeq \frac{7.8 \times 10^{-3}}{n^3} \\ &\quad \times \left\{ \frac{\lambda M^n}{M_{\text{Pl}}^4} \left(\frac{n}{2} [(n+3)\mathcal{N} + n] \right)^{(n+1)} \right\}^{3/(n+3)}, \end{aligned} \quad (45)$$

$$n_s - 1 \simeq -\frac{3(n+1)}{n} \epsilon \simeq -\frac{3(n+1)}{(n+3)\mathcal{N} + n}. \quad (46)$$

The scalar-to-tensor ratio is given by

$$r \simeq \frac{64}{3} \left(\frac{2}{3} \right)^{1/2} \epsilon \simeq \frac{17n}{(n+3)\mathcal{N} + n}. \quad (47)$$

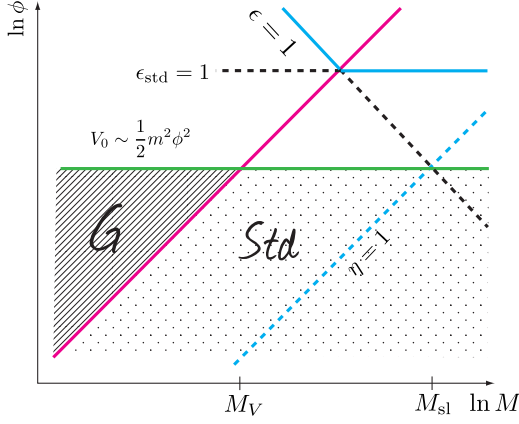


FIG. 2: The same diagram as Fig. 1, but for new and hybrid inflation with $|\eta_{\text{std}}| < 1$. The Galileon effect operates above the magenta line, the slow-roll condition $\epsilon < 1$ is satisfied below the cyan line, and the constant piece V_0 dominates the potential below the green line.

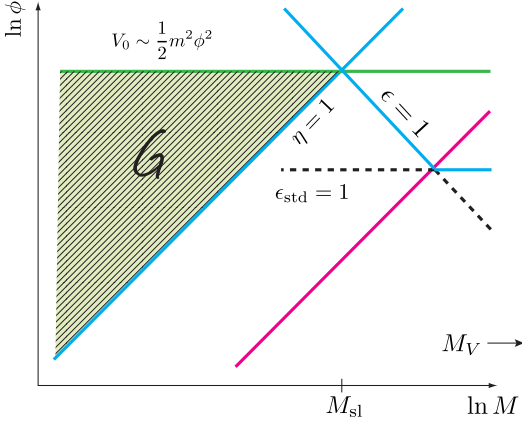


FIG. 3: The same diagram as Fig. 1, but for new and hybrid inflation with $|\eta_{\text{std}}| > 1$. The slow-roll conditions are satisfied below the $\epsilon = 1$ line and above the $\eta = 1$ line. Therefore, even though $\eta_{\text{std}} > 1$, G-inflation can occur in the green, shaded region.

Except for the special case with $M \sim M_{\text{sl}}$ we may have $\phi_V \gg \phi_{\eta G} \mathcal{N}^2$. If this is satisfied then we find

$$\phi_{\mathcal{N}} \simeq \phi_V. \quad (61)$$

Now we investigate the primordial perturbation. In this case, we have

$$\mathcal{F} \simeq \frac{4m\phi^{1/2}}{3M^{3/2}}, \quad \mathcal{G} \simeq \frac{2m\phi^{1/2}}{M^{3/2}}. \quad (62)$$

From Eq. (32) the power spectrum of the primordial density perturbation generated during the potential-dominated new G-inflation is given by

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{\sqrt{6}}{32\pi^2} \frac{V_0^3}{m^3 M_{\text{Pl}}^6 M^{3/2} \phi^{3/2}}. \quad (63)$$

More concretely, one can evaluate as

$$\mathcal{P}_{\mathcal{R}} \simeq 4.6 \times 10^{-3} \frac{V_0^{9/4}}{m^{3/2} M_{\text{Pl}}^6 M^{3/2}}, \quad (64)$$

$$n_s - 1 \simeq -6\epsilon + 3\tilde{\eta} \simeq -6.3 \times \frac{m^{3/2} M_{\text{Pl}}^2 M^{3/2}}{V_0^{5/4}}, \quad (65)$$

$$r \simeq \frac{64}{3} \left(\frac{2}{3}\right)^{1/2} \epsilon \simeq 14 \times \frac{M_{\text{Pl}}^2 m^{3/2} M^{3/2}}{V_0^{5/4}}. \quad (66)$$

C. Hybrid inflation

Finally, let us study the hybrid inflation model [22] where the potential is effectively given by

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2. \quad (67)$$

We consider the range $\phi > 0$, and hence take $g = +1/M^3 > 0$. Hybrid inflation ends when the waterfall field becomes tachyonic. Let ϕ_{tac} be the value of ϕ where this occurs. We will therefore focus on the range $\phi_{\text{tac}} < \phi < \sqrt{2V_0}/m$. For $\phi \gg \sqrt{2V_0}/m$, the constant piece V_0 in the potential can be ignored, and hence the situation reduces to chaotic inflation studied in Sec. III A.

The situation here is analogous to the case of new inflation. The Galileon effect operates for $\phi \gg \phi_G$, where ϕ_G is given in Eq. (51). The slow-roll parameters for $\phi > \phi_G$ are given by

$$\epsilon = \frac{M_{\text{Pl}}^2 m^3 M^{3/2} \phi^{3/2}}{2V_0^2}, \quad \tilde{\eta} = \frac{M_{\text{Pl}}^2 m M^{3/2}}{2V_0 \phi^{1/2}}, \quad (68)$$

so that the slow-roll conditions are satisfied in the range $\phi_{\eta G} < \phi < \phi_{\epsilon G}$, where $\phi_{\eta G}$ and $\phi_{\epsilon G}$ are the quantities defined in Eq. (54). Thus, the inflaton dynamics can be summarized in Figs. 2 and 3, depending on the values of M_V and M_{sl} . Since $\phi < 0$, G-inflation is followed by standard hybrid inflation if $M < M_V < M_{\text{sl}}$. If $M < M_{\text{sl}} < M_V$ then G-inflation can occur even though standard inflation would be impossible as $\tilde{\eta}_{\text{std}} > 1$. We remark that hybrid inflation ends at $\phi = \phi_{\text{tac}}$, which is not indicated explicitly in the figures, and for $\phi > \sqrt{2V_0}/m$ hybrid inflation reduces to chaotic inflation, which is not shown in the figures either.

The primordial perturbation is described in the same way as in new inflation,

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{\sqrt{6}}{32\pi^2} \frac{V_0^3}{m^3 M_{\text{Pl}}^6 M^{3/2} \phi^{3/2}}. \quad (69)$$

The number of e -folds during hybrid G-inflation \mathcal{N} reads

$$\mathcal{N} = \int_{\phi}^{\phi_{\text{tac}}} \frac{H}{\dot{\phi}} d\phi \simeq \frac{2V_0}{M_{\text{Pl}}^2 M^{3/2} m} \left(\phi^{1/2} - \phi_{\text{tac}}^{1/2} \right). \quad (70)$$

We then obtain the field value evaluated \mathcal{N} e -folds before the end of inflation as,

$$\phi_{\mathcal{N}} = \left(\phi_{\text{tac}}^{1/2} + \frac{M_{\text{Pl}}^2 M^{3/2} m}{2V_0} \mathcal{N} \right)^2. \quad (71)$$

In the case where

$$\phi_{\text{tac}} \gg \left(\frac{M_{\text{Pl}}^2 M^{3/2} m}{2V_0} \mathcal{N} \right)^2, \quad (72)$$

we have

$$\phi_{\mathcal{N}} \simeq \phi_{\text{tac}}, \quad (73)$$

and thus

$$\mathcal{P}_{\mathcal{R}} \simeq 7.6 \times 10^{-3} \frac{V_0^3}{m^3 M_{\text{Pl}}^6 M^{3/2} \phi_{\text{tac}}^{3/2}}, \quad (74)$$

$$n_s - 1 \simeq 3\tilde{\eta} \simeq \frac{3M_{\text{Pl}}^2 m M^{3/2}}{2V_0 \phi_{\text{tac}}^{1/2}}, \quad (75)$$

$$\begin{aligned} r &\simeq \frac{64}{3} \left(\frac{2}{3} \right)^{1/2} \epsilon \\ &\simeq 8.7 \times \frac{M_{\text{Pl}}^2 m^3 M^{3/2} \phi_{\text{tac}}^{3/2}}{V_0^2}. \end{aligned} \quad (76)$$

In the opposite case where

$$\phi_{\text{tac}} \ll \left(\frac{M_{\text{Pl}}^2 M^{3/2} m}{2V_0} \mathcal{N} \right)^2, \quad (77)$$

we have

$$\phi_{\mathcal{N}} \simeq \frac{M_{\text{Pl}}^4 M^6 m^2}{4V_0^2} \mathcal{N}^2, \quad (78)$$

so that

$$\mathcal{P}_{\mathcal{R}} \simeq 6.2 \times 10^{-3} \frac{V_0^6}{M_{\text{Pl}}^{12} m^6 M^6 \mathcal{N}^3}, \quad (79)$$

$$n_s - 1 \simeq \frac{3}{\mathcal{N}}, \quad (80)$$

$$r \simeq 1.1 \times \frac{M_{\text{Pl}}^8 m^6 M^6}{V_0^5} \mathcal{N}^3. \quad (81)$$

IV. HIGGS G-INFLATION

Let us now construct a Higgs inflation model in the context of G-inflation. The tree-level SM Higgs Lagrangian is

$$S_0 = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - |D_\mu \mathcal{H}|^2 - \lambda(|\mathcal{H}|^2 - v^2)^2 \right], \quad (82)$$

where D_μ is the covariant derivative with respect to the SM gauge symmetry, \mathcal{H} is the SM Higgs boson, v is the

vacuum expectation value (VEV) of the SM Higgs and λ is the self coupling constant. Since we would like to have a chaotic inflation-like dynamics of the Higgs boson, we consider the case where its neutral component $\phi := \sqrt{2}|\mathcal{H}_0|$ is very large compared with to v : $\phi \gg v$. In this situation, we have only to consider a simpler action,

$$S_0 = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda}{4} \phi^4 \right]. \quad (83)$$

In addition to the above action, we consider a Galileon-type interaction, which breaks Galilean shift symmetry weakly,

$$\begin{aligned} S_G &= \int d^4x \sqrt{-g} \left[-\frac{2\mathcal{H}^\dagger}{M^4} D_\mu D^\mu \mathcal{H} |D_\mu \mathcal{H}|^2 \right] \\ &\rightarrow \int d^4x \sqrt{-g} \left[-\frac{\phi}{2M^4} \square \phi (\partial_\mu \phi)^2 \right], \end{aligned} \quad (84)$$

where M is a mass parameter. Here we assume $M > 0$. Note that gauge fields that couples to ϕ receive heavy mass from the field value of the Higgs boson and hence we can neglect the effect of gauge fields when we consider the inflationary trajectory. This setup corresponds to the case

$$K = X - V(\phi), \quad V(\phi) = \frac{\lambda}{4} \phi^4, \quad G = -\frac{\phi}{M^4} X. \quad (85)$$

The Galileon effect operates provided that $gV' \gg 1$, *i.e.*,

$$\phi \gg \lambda^{-1/4} M. \quad (86)$$

In this regime the slow-roll parameters are given by

$$\epsilon = \frac{8M_{\text{Pl}}^2 M^2}{\lambda^{1/2} \phi^4} = \frac{4}{3} \tilde{\eta} = -2\alpha, \quad (87)$$

and $\beta = 0$, so that the slow-roll conditions are satisfied if

$$\phi \gg 2^{3/4} \lambda^{-1/8} M_{\text{Pl}}^{1/2} M^{1/2} =: \phi_{\text{end}}. \quad (88)$$

From Eqs. (86) and (88) one can define a mass scale, analogously to Eq. (41),

$$M_c := \lambda^{-3/4} M_{\text{Pl}}. \quad (89)$$

If $M \ll M_c$, Higgs G-inflation proceeds even if standard Higgs inflation would otherwise be impossible. One can draw essentially the same diagram as Fig. 1 for Higgs-G inflation.

The number of e -folds \mathcal{N} is given by

$$\mathcal{N} = \int_{\phi}^{\phi_{\text{end}}} \frac{H}{\dot{\phi}} d\phi = \frac{1}{16} \frac{\lambda^{1/2}}{M_{\text{Pl}}^2 M^2} \phi^4 - \frac{1}{2}, \quad (90)$$

from which we obtain the field value evaluated \mathcal{N} e -folds before the end of inflation as

$$\phi_{\mathcal{N}} = (16\mathcal{N} + 8)^{1/4} \lambda^{-1/8} M_{\text{Pl}}^{1/2} M^{1/2}. \quad (91)$$

As will be mentioned at the end of this section, reheating after Higgs G-inflation proceeds in the same way as in the standard inflationary models, and hence the history of the Universe after inflation will not be altered. Therefore, we use the value $\mathcal{N}_{\text{COBE}} = 60$.

Now let us turn to the primordial perturbation in this model. Using the slow-roll approximation, we obtain

$$\mathcal{F} \simeq \frac{4\lambda^{1/2}\phi^2}{3M^2}, \quad \mathcal{G} \simeq \frac{2\lambda^{1/2}\phi^2}{M^2}. \quad (92)$$

We thus arrive at

$$\begin{aligned} \mathcal{P}_{\mathcal{R}} &= \frac{1}{4\pi^2} \frac{H^4}{\dot{\phi}^2 c_s^2 \sqrt{\mathcal{F}\mathcal{G}}} \\ &= \frac{(2\mathcal{N}_{\text{COBE}} + 1)^2 \lambda^{1/2}}{8\pi^2} \left(\frac{3}{8}\right)^{1/2} \left(\frac{M}{M_{\text{Pl}}}\right)^2 \\ &\simeq 1.1 \times 10^2 \lambda^{1/2} \left(\frac{M}{M_{\text{Pl}}}\right)^2, \end{aligned} \quad (93)$$

and

$$n_s = 1 - 4\epsilon \simeq 1 - \frac{4}{2\mathcal{N}_{\text{COBE}} + 1} \simeq 0.967. \quad (94)$$

According to WMAP observations, $\mathcal{P}_{\mathcal{R}} = 2.4 \times 10^{-9}$ [5], and hence

$$M \simeq 4.7 \times 10^{-6} \lambda^{-1/4} M_{\text{Pl}} \simeq 10^{13} \text{ GeV}. \quad (95)$$

The scalar-to-tensor ratio is given by

$$r = \frac{64}{3} \left(\frac{2}{3}\right)^{1/2} \epsilon \simeq 17 \times \frac{1}{2\mathcal{N}_{\text{COBE}} + 1}. \quad (96)$$

For $\mathcal{N}_{\text{COBE}} = 60$ this yields $r \simeq 0.14$, which is large enough to be detected by the forthcoming observation by PLANCK [23].

Note that in the above discussion we have neglected the quantum corrections. In order to have the precise relation between the potential of the Higgs field and the observational signatures such as the tensor-to-scalar ratio r , we must know/assume the complete theory valid up to the inflationary scale, which is left for future study. However, the qualitative argument will not be changed even if we take into account quantum effects, because it can be absorbed by the variation of M .

Before closing this section, let us take a brief look at reheating after Higgs G-inflation. The dynamics of the inflaton field during the reheating stage is non-trivial in general when one considers non-standard kinetic terms. In the present case, however, the effect of Galileon-like interaction can safely be ignored during reheating because $g = \phi/M^4$ is suppressed around the minimum of the potential, $\phi \ll M$. In Fig. 4 we show a numerical example of the evolution of ϕ and ρ in the final stage of Higgs G-inflation and in the beginning of the reheating stage, when the Higgs field oscillates rapidly. We have confirmed that the Galileon terms become ineffective very soon after inflation ends, leading to reheating in the same way as in the usual case, $\rho \propto a^{-4}$, for the quartic potential.

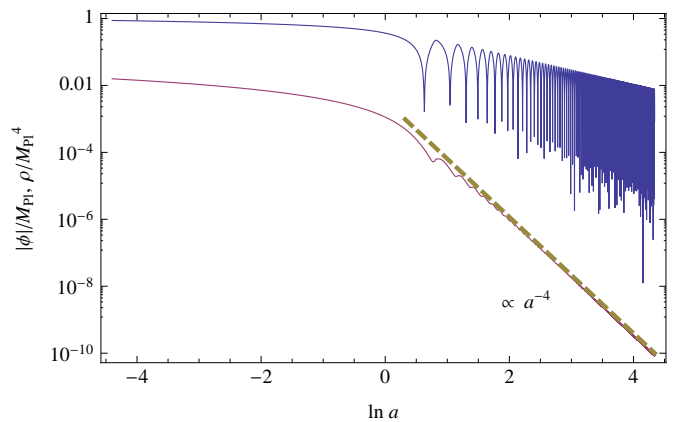


FIG. 4: The evolution of $|\phi|$ (blue, oscillating line) and ρ (purple line) in the very final stage of Higgs G-inflation and in the reheating stage thereafter. We set $a(t_{\text{end}}) = 1$ with $\epsilon(t_{\text{end}}) = 1$. The parameters are given by $\lambda = 0.1$ and $M = 0.01 \times M_c$.

V. DISCUSSION

The Galileon-like nonlinear derivative interaction, $G(\phi, X)\Box\phi$, opens up a new arena of inflation model building while keeping the models healthy, and the novel class of inflation thus developed —*G-inflation*— possesses various interesting aspects to be explored. In [15] the extreme case was emphasized where G-inflation is driven purely by the kinetic energy, though the background and the perturbation equations have been derived without assuming any specific form of $K(\phi, X)$ and $G(\phi, X)$. In this paper, we have studied the effects of the Galileon term on potential-driven inflation. Although the energy density is dominated by the potential anyway, the dynamics of the inflaton is nontrivial when the $G(\phi, X)\Box\phi$ term participates more dominantly than the usual linear kinetic term X . We have demonstrated that the Galileon term makes the potential effectively flatter so that slow-roll inflation proceeds even if the potential is in fact too steep to support conventional slow-roll inflation. In light of this fact, we have constructed a viable model of Higgs inflation, *i.e.*, Higgs G-inflation, showing that the power spectrum of the primordial density perturbation is compatible with current observational data. The tensor-to-scalar ratio r is large enough to be detected by the Planck satellite.

In this paper we have focused on the power spectrum of the curvature perturbation and the consistency relation relative to the amplitude of tensor perturbations. Primordial non-Gaussianity would be another powerful probe to discriminate G-inflation among others. Unfortunately, however, non-Gaussianity arising from the present potential-driven models is estimated to be not large. This is because f_{NL} is composed of $(1 - 1/c_s^2)$ and slow-roll suppressed terms with $c_s^2 \simeq 2/3$ in the present model, leading to $f_{\text{NL}} \lesssim \mathcal{O}(1)$. (Detailed computation of

primordial non-Gaussianity from G-inflation will be given elsewhere [24]; see also [16–18].) We would thus conclude that the smoking gun of potential-driven G-inflation is the consistency relation which is unique enough to distinguish G-inflation from standard canonical inflation and k-inflation.

We have focused on the (generalized form of the) leading order Galileon term. As demonstrated in [14], higher order terms play an important role in the cosmological dynamics of Galileon dark energy models. Therefore, it would be interesting to consider the effects of the higher order Galileon terms in the context of primordial inflation, which is left for further study.

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